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The Impact of Crown Fire and Clear-cut Logging on Lowland Black Spruce Ecosystem Dynamics (II)

Development of Growth and Yield Models for Lowland Black Spruce Stands

Submitted to

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EXECUTIVE SUMMARY

As a part of a larger project, stand information and stem analysis data were collected from 37 black spruce plots (30-191 years old) growing on lowland organic soils within Manitoba Model Forest area. The data were used to develop growth and yield models, including individual tree models, height/site index models, and stand volume models. The study not only filled a data gap of lowland black spruce growth and yield, but also provided basic methodologies to forest growth/yield studies in Manitoba.

Individual tree models were developed based on the 143 stem analysis trees sampled in the study. These models were:

1. the height-diameter model that can be used to predict total height from breast height diameter;
2. the stump-breast height diameter model that can be used to predict stump diameter from breast height diameter or vise versa;
3. the diameter inside/outside model that can be used to predict diameter inside bark from diameter outside bark or vise versa;
4. the tape model that can be used to predict diameter at any height of a tree stem from its breast height diameter and total height;
5. the total volume models that can be used to predict total volume of a tree either from its breast height diameter or from its breast height diameter and total height;
6. the merchantable volume models that can be used to predict merchantable volume of a tree (defined according to either Manitoba Forestry Branch's or Pine Falls Paper Company's criteria) from its total volume; and
7. the volume inside/outside bark model that can be used to predict the volume inside bark from volume outside bark or vise versa.

A merchantable volume model used by Manitoba Forestry Branch and a total volume model used by Repap Manitoba were tested against our data. Testing results indicated that both models underestimated volume by 11.7 to 14.8%. Further residual analysis suggested that there was a lack of fit in both models. Therefore, there is a need to improve these models or develop new models. On the request of Pine Falls Paper Company, the volume saved by using a stump height of 17.5 cm instead of 30 cm was calculated. The saved volume was between 1.5 to 5.5% of the merchantable volume (using 30 cm stump height) depending on the size of the tree. A model that can predict the saved volume from breast height diameter was developed.

Stem analysis data were obtained from three dominant trees sampled on each of the 22 plots (with breast height age $\geq$ 50 years). These data were used to develop height and site index models. The height model can be used to predict mean height of dominant trees at any age given a site index while the site index model can be used to predict site index given any pair of height and breast height age.

Both total and merchantable stand volume models were developed for lowland black spruce stands. The total stand volume models were developed based on all 37 plots while the merchantable volume models were developed based on those 22 plots with actual site index observations. The total stand volume models can be used to predict the total volume of a stand either from basal area or from the product of basal area and mean stand height. The merchantable stand volume models can be used to predict the merchantable volume of a stand from basal area, site index, and stand age at breast height.

Since all the models developed in the study were based on a relatively small data set obtained from one geographical region (i.e., Manitoba Model Forest area) and one site type (i.e., organic soils), it is advised that these models may not necessarily apply to other regions or other site types. Independent testing is recommended should such an application be required. Given the wide ecological amplitude of black spruce, we recommend future study should be conducted in the same study area to collect data from black spruce stands associated with other types of soils. The new data together with the data obtained from this study as well as other sources (e.g., temporary or permanent sample plots from provincial forest resource inventory) would provide a large database for developing growth/yield models that can be applied to black spruce growing on any site conditions within the study area. Considering the importance of the species to the province, we further recommend that more data should be collected across the province in order to develop ecologically-based growth/yield models that can be applied to any site type in any geographical region. Such a study may need coordinated effort between government agencies and industries.

TABLE OF CONTENT

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1. INTRODUCTION

Black spruce [*Picea mariana* (Mill.) B.S.P.] is one of the most prominent and widely distributed tree species in boreal forests of North America. It grows on various soil types from deep humus through clays, loams, sands, coarse till, boulder pavements, and shallow mantles over bedrock (Burns and Honkala 1990). On wet organic soils, black spruce forms virtually pure stands, which are commonly even-aged and initiated after fire. Black spruce stands may, however, become uneven-aged through eventual deterioration of the original stands that provide openings for reproduction of its own (Perala 1971). Black spruce is abundant across Manitoba. In the study area, black spruce appears as the primary or secondary species in 80% of the stands according to the 1986 Forest Resource Inventory. Black spruce also consists of a significant portion of timber supply of Pine Falls Paper Company (Keenan, pers. comm.). Despite the importance, growth and yield models have not been developed for black spruce in Manitoba.

Being able to predict growth and yield is fundamental to develop any timber management plan, and timber management is an integral part of ecosystem-based forest management. Most ecosystem models require forest growth/yield information as essential input. Such information, however, has been lacking in Manitoba. Therefore, it has been decided that growth/yield data should be collected along with ecological data when conducting the project of "the impact of fire and logging on lowland black spruce ecosystem dynamics" (Project #95-2-26 of Manitoba Model Forest Inc.). It is expected that these data could be used to construct growth/yield models for lowland black spruce stands, thus provide essential data for foresters to run timber supply as well ecosystem models.

2. RESEARCH OBJECTIVES

The project was designed to investigate ecosystem dynamics following a crown fire and a clear-cut logging on lowland black spruce ecosystems. The overall objective was to examine how fire and logging differ in affecting vegetation, soils, and soil seedbanks of lowland black spruce stands. The project consists of four components: (1) regeneration of trees and understory plants, (2) growth and yield, (3) soils, and (4) seedbanks. The current report only addressed component (2) of the project. The component (1) of the project has been reported by Wang and Harrison (1997). The components of soils and seedbanks will be reported by Drs. L. Fuller and R.J. Staniforth, respectively.

The main objective of this report was to develop growth and yield models for lowland black spruce stands. Specifically, we developed (1) various types of individual tree models, (2) height/site index models, and (3) stand yield models. In addition, two individual tree volume models that have been used by Manitoba Forestry Branch and Repap Manitoba Inc. were tested using the data collected in the study. Models used by Pine Falls Paper Company was not available at this time, thus was not included in the testing.

3. MATERIALS AND METHODS

3.1. Study Sites

The procedure of site selection followed Wang and Harrison (1997). Of all 71 plots sampled, only those 37 plots with closed canopy (> 30 years old) were included in this study. These stands ranged from 30 - 191 years old, and were either black spruce-dominated or pure black spruce. The location of the study sites and a brief description of climate, topography, soil and vegetation of the study area is given by Wang and Harrison (1997). The term "lowland" in this study refers specifically to those sites with organic soils as defined by CSSC (1978). They are commonly associated with wet, poorly drained, low-lying or depression conditions. Lowland black spruce stands are among the most commonly encountered forest types in the study area as well as in Manitoba's boreal forests. Their wet and deep (>40 cm) organic soils, often with hummock-hollow microtopography, are characterized by a continuous ground cover of feathermoss or sphagnum moss. Soils are classified as Fibrisols, Mesisols, or less frequently Humisols. According to Manitoba's forest ecosystem classification (Zoladeski *et al.* 1995), lowland black spruce stands consist of vegetation types of V30, V31, V32, and V33 and soil types of S12F and S12S. Detailed description on these vegetation and soil types is given by Zoladeski *et al.* (1995).

3.2. Data Collection

3.2.1. Stand information

Stand information (including any trees taller than 2 m) was collected on each plot. Each tree was identified into species. The diameter at breast height (dbh) was measured for every tree present within the 10x10 m plot. The diameter was recorded into dbh classes, which were defined at 2.0 cm intervals starting from 1.0 cm. The first dbh class included all trees with dbh's between 1.0 cm and 2.9 cm, the second included those between 3.0 cm and 4.9 cm, and so forth. For each tree species and each dbh class, at least one tree was randomly selected to determine breast height age (bha) and height. On each selected tree, a core was removed at breast height (1.3 m from the base) for ring counting later in the lab. The number of rings represent the age of the tree at that particular height (i.e., bha). The total height of each selected tree was measured using a clinometer.

3.2.2. Stem analysis

Stem analysis followed a similar procedure described by Wang *et al* (1994). On each plot, three dominant black spruce trees were selected and felled. On some plots, one tree for each 2 cm dbh class was also selected and felled. For each felled tree, the total height, dbh, length of live crown, diameter at the base of the living crown were measured in the field. Annual height growth for the last five years identifiable through growth whorl was also measured in the field for each felled tree. Stem disks were cut at 0, 0.3, 0.6, and 1.3 m and then were taken at 1 m intervals between 1.3 m and the top of each tree. Each disk was labeled accordingly, and taken to the lab where their diameters were measured, and their rings were counted.

On each disk, rings were counted with the assistance of a binocular microscope. Diameters (inside and outside bark) at two direction (N-S and E-W) were measured to 0.01 cm. Height and age data obtained from stem analyses can be biased if the height of the cross-cut is taken as the tree height for the age obtained from ring counting. This is because of the presence of a 'hidden tip' above the cross-cut (Carmean 1972). Therefore, the raw height and age data were adjusted using Carmean's (1972) algorithm to calculate tree height corresponding to the age at each cross-cut (recommended by Dyer and Bailey 1987). In total 143 trees were sectioned, and about 2,000 disks were obtained.

For each sectioned tree, the volume of each section was calculated as:

[3.1] $$V_i = \frac{1}{80000}\pi\left(d_{bot}^2 + d_{top}^2\right)H_i$$

where

V_i = volume (m^3) of section i ($i = 1, 2, 3, \ldots n$); $V_1 = V_{stp}$ (stump volume)
d_{bot} = diameter (cm) at the bottom of the section
d_{top} = diameter (cm) at the top of the section
H_i = length of the section.
$\pi \cong 3.1416$.

The tip of each sectioned tree was assumed as a canoid, and calculated as follows:

[3.2] $$V_t = \frac{1}{3}\pi(d/200)^2 h_t$$

where

V_t = volume (m^3)of the tip

d = diameter at the base of the tip

h_t = the length of the tip

$\pi \cong 3.1416$.

Total volumes (both inside and outside bark) was calculated as:

[3.3] $$V = \sum_{i=1}^{n} V_i + V_t$$

where

V = total volume (m^3)

V_i = volume of section i ($i = 1, 2, 3, \dots n$)

V_t = volume of the tip (m^3).

3.3. Data Analysis

Scatterplots with locally weighted smoothing (Wilkinson 1990) curves were used to explore relationships between two variables. Based on this exploration, appropriate models were selected to fit the data. All statistical analyses were performed using SYSTAT for Windows (version 5.1) (Wilkinson 1990). For nonlinear model, R^2 reported in the study is the corrected R^2; for linear model, R^2 reported in the study is the adjusted R^2 (Wilkinson 1990).

3.3.1. Individual tree models

1. The height-diameter model

Richards' three parameter model was found appropriate for lowland black spruce:

[3.4] $$H = a(1 - e^{-bD})^c$$

where

H = total height (m)

D = diameter at breast height (cm) outside bark

e = base of the natural logarithm ($\cong 2.71828$)

a, b, c = parameters to be estimated.

2. The stump-breast height diameter model

A simple linear equation was used to quantifying the relationship between the stump (30 cm from the ground) diameter and breast height (1.3 m from the ground) diameter:

$$DOB_{stp} = a + bD \qquad [3.5]$$

where

DOB_{stp} = diameter outside bark at 0.30 m stump height

D = diameter at breast height outside bark

a, b = parameters to be estimated.

3. The diameter inside/outside model

A simple linear equation was used to express diameter inside bark as a function of diameter outside bark:

$$DIB = a + bDOB \qquad [3.6]$$

where

DIB = diameter inside bark at any point on the stem

DOB = corresponding diameter outside bark

a, b = parameters to be estimated.

4. The tape model

Based on evaluation of existing tape equations, Huang (1994) found that the variable-exponent tape equation (Kozak 1988) was appropriate for major Alberta tree species. Therefore, the variable-exponent tape equation was selected in the study:

$$d = a_0 D^{a_1} {a_2}^{D} X^{b_1 Z^2 + b_2 \ln(z=0.001) + b_3 \sqrt{Z} + b_4 e^{z} + b_5 (D/H)} \qquad [3.7]$$

where

$X = (1 - \sqrt{h/H}) / (1 - \sqrt{p})$

d = diameter inside bark (cm) at h

h = height above ground (m)

H = total tree height

D = diameter at breast height outside bark (cm)

$Z = h/H$

p = location of the inflection point, assumed to be at 22.5% of the total height above the ground

ln = the natural logarithm

e = base of the natural logarithm ($\cong$ 2.71828)

$a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5$ = parameters to be estimated.

Merchantable volume of each tree was calculated for those stem analysis trees that are merchantable in size:

[3.8] $$V_m = V - V_s - V_t$$

where

V_m = merchantable volume (m^3)

V = total volume (m^3)

V_s = stump volume (m^3) at stump height of 30 cm

V_t = tip volume (m^3, volume from top diameter to the tip of the tree).

Stump volume in the study equal the volume of the first section, which was calculated from Eq. [3.1]. Eq. [3.7] was used to calculate tree height at top diameter (*h*) through iteration procedure. Tip volume was then calculated using Eq. [3.2].

Two merchantable volumes of each sectioned tree were calculated according to the criteria from Manitoba Forestry Branch (minimum tree diameter = 9.1 cm; top diameter = 7.6 cm; stump height = 30 cm) and Pine Falls Paper Company (minimum tree diameter = 11.1 cm; top diameter = 6.3 cm; stump height = 30 cm). On the request of Pine Falls Paper Company, the volume saved by harvesting at 17.5 cm, instead of 30 cm, stump height was also calculated using tape equation (Eq. [3.7]) and stem analysis data.

5. The volume model

Richards' model was found appropriate to expressing individual tree volume either as a function of breast height diameter (Eq. [3.9]) or as a function of the product of squared breast height diameter and total height of the tree (Eq. [3.10]):

[3.9] $$V_{OB} = a(1 - e^{-bD})^c$$

where

V_{OB} = total volume (m^3) outside bark

D = breast height diameter outside bark (cm)

e = base of the natural logarithm ($\cong 2.71828$)

a, b, c = parameters to be estimated.

[3.10] $$V_{OB} = a(1 - e^{-b\frac{D^2H}{10000}})^c$$

where

V_{OB} = total volume (m^3) outside bark

D = breast height diameter outside bark (cm)

H = total tree height (m)

e = base of the natural logarithm ($\cong 2.71828$)

a, b, c = parameters to be estimated.

A simple linear model was found appropriate to predict volume inside bark from volume outside bark:

[3.11] $$V_{IB} = a + bV_{OB}$$

where

V_{IB} = total volume (m^3) inside bark

V_{OB} = total volume (m^3) outside bark

a, b = parameters to be estimated.

A linear relationship was also used to quantifying the relationship between merchantable volume (inside bark) and total volume (outside bark):

[3.12] $$V_M = a + bV_{OB}$$

where

V_{OB} = total volume (m^3) outside bark

V_M = merchantable volume (m^3) inside bark

a, b = parameters to be estimated.

6. Testing existing individual tree volume models

To test existing individual tree volume equations, we obtained equations used by Manitoba Forestry Branch (Eq. [3.13]) and Repap Manitoba Inc. (Eq. [3.14])

[3.13] $$V_M = -0.010307 + 0.003361824\frac{D^2 H}{100}$$

Where

V_M = merchantable volume (m^3) outside bark
D = breast height diameter outside bark (cm)
H = total tree height (m).

[3.14] $$V_{OB} = \frac{D^2}{361.8043282 + 23150.31513 / H}$$

where

V_{OB} = total volume (m^3) outside bark
D = breast height diameter outside bark (cm)
H = total tree height (m).

The volume prediction from Eq. [3.13] was compared with volume estimation from Eq. [3.8]. The volume prediction from Eq. [3.14] was compared with the volume estimated through the stem analysis procedure (Eq. [3.3]).

3.3.2. Height/site index models

Graphs of height versus age were examined for each site tree. If suppression or damage was apparent, data from the site tree was deleted or truncated. An average height growth curve was determined for each plot from the individual tree stem analysis data using Richards' (1959) three-parameter model

[3.15] $$H = 1.3 + b_1(1 - e^{-b_2 A})^{b_3}$$

where

H = height (m)
A = age (years) at breast height
e = base of the natural logarithm
b_1, b_2, b_3 = parameters to be estimated for each stand.

As a result, within-plot standard errors of estimates for the above model in the 37 plots averaged 0.66 m, with a standard deviation of 0.28 m.

Actual site index in each stand was determined from the model by setting A = 50 years (site index age at breast height). The model was evaluated for each stand at every decade from age 10 to the decadal age nearest the age of the oldest tree in that stand to provide the database used for constructing height growth curves. As a result, 266 decadal observations of height, age, and site index for the 22 plots were produced. Of these, 192 observations between age 10 and 100 from 22 plots were used to develop height growth models which required site index as a predictor. The remaining 15 plots, which had average b.h. age < 50 years, were not used to develop height/site index model as site indices of those plots can not be reliably extrapolated.

Ek-Payandah's (Ek 1971, Payandeh 1974) was selected to fit the data:

[3.16] $$H = 1.3 + b_1 (SI^{b_2})(1 - e^{-b_3 A})^{b_4 SI^{b_5}}$$

where

H = total tree height (m)

SI = Site index (m)

A = breast height age (years)

e = base of the natural logarithm

b_1, b_2, b_3, b_4, b_5 = parameters to be estimated for each stand.

The above model has also been fitted to black spruce data from site index tables (Payandeh 1977) and stem analysis (Payandeh 1978) in Ontario.

3.3.3. Stand volume models

Total stand volume of each plot was calculated by applying individual tree volume equation (Eq. [3.10]) to each dbh class. The calculated stand volume (m^3/plot) was converted to m^3/ha for each plot, and then related to various stand attributes. The following two equations were found appropriate to predict current yield (total stand volume):

[3.17] $$Y = e^{a + b\ln(BA)}$$

where

Y = total stand volume (m^3/ha)

BA = basal area (m^2/ha)

e = base of the natural logarithm ($\cong 2.71828$)

ln = the natural logarithm

a, b = parameters to be estimated.

[3.18] $$Y = a(1 - e^{-b(BAxHT)})^c$$

where

Y = total stand volume (m^3/ha)

BA = basal area (m^2/ha)

HT = mean stand height (m)

a, b, c = parameters to be estimated.

Two merchantable stand volume (defined by Manitoba Forestry Branch and Pine Falls Paper Company, respectively) of each plot were calculated by applying Eqs. [3.10] and [3.12]. The calculated merchantable volume (m^3/plot) was converted to m^3/ha for each plot, and then related to various stand attributes. The following equations was found appropriate to predict merchantable stand volume:

[3.19] $$MSV = e^{b_0 + b1\ln(BA) + b2\ln(SI) + b3/A}$$

where

MSV = merchantable stand volume (m^3/ha)

BA = basal area (m^2/ha)

SI = site index (m)

A = breast height age (year)

e = base of the natural logarithm ($\cong 2.71828$)

ln = the natural logarithm

b_0, b_1, b_2, b_3 = parameters to be estimated.

The same type of equation was used to predict merchantable volume of lowland black spruce stands in Minnesota (Perala 1971).

4. RESULTS AND DISCUSSIONS

4.1. Individual Tree Models

A summary of selected attributes of those trees sampled in the study is given in Table 4.1. Since all the individual tree models were developed based on these data, it is advised that the resulted model should not be applied beyond the range of the data without independent testing.

Table 4.1. Summary of selected attributes of lowland black spruce trees sampled in the study (n = 143).

Stem attribute	Range	Mean	SD	CV
Total height (m)	3.17 - 20.70	11.25	4.33	0.38
DBH (cm)	2.45 - 23.95	11.35	4.74	0.42
Volume (m3)	0.0014 - 0.3836	0.0841	0.0788	0.94

SD - standard deviation.
CV - coefficient of variation.

4.1.1. The height-diameter model

Height was found to be best predicted by breast height diameter (Eq. [4.1]):

[4.1] $$H = 39.57(1 - e^{-0.027908D})^{1.015651}$$

$R^2 = 0.85$ $\quad$ $RMSE = 1.68\ (m)$ $\quad$ $n = 143$

where

H = total height (m)
D = diameter at breast height outside bark (cm)
e = base of the natural logarithm ($\cong 2.71828$)
$RMSE$ = root mean squared error.

In the case of lacking height measurements, Eq. [4.1.] can be used to predict tree height from field measurement of tree diameter at breast height outside bark. The same type of model was also used by Huang and Titus (1994).

4.1.2. The stump-breast height diameter model

Significant relationship was found between the stump (30 cm from the ground) diameter and breast height (1.3 m from the ground) diameter (Eq. [4.2]).

[4.2] $$DOB_{stp} = 0.424206 + 1.118146D$$

$$R^2 = 0.984 \qquad SEE = 0.679 \text{ (cm)} \qquad n = 143$$

where

DOB_{stp} = diameter outside bark at 0.30 m stump height (cm)

D = diameter at breast height outside bark (cm)

SEE = Standard error of estimate.

The regression model is commonly used for conversion between these two diameters. It can be used to predict stump diameter from field measurement of breast height diameter, or to predict breast height diameter from field measurements of stump diameter. For the latter, equation [4.2] can be rearranged as follows:

[4.3] $$D = \frac{DOB_{stp} - 0.424206}{1.118146}$$

where

D = diameter at breast height outside bark (cm)

DOB_{stp} = diameter outside bark at 0.30 m stump height (cm).

4.1.3. The diameter inside/outside model

A significant relationship was found between diameters inside and outside bark (Eq. [4.4]):

[4.4] $$DIB = -0.234 + 0.977DOB$$

$$R^2 = 0.999 \qquad SEE = 0.177 \text{ (cm)} \qquad n = 143$$

where

DIB = diameter inside bark at any point on the stem (cm)

DOB = Corresponding diameter outside bark (cm)
SEE = Standard error of estimate.

A relationship between diameters inside and outside barks is commonly used for conversion between these two diameters. The relationship can be used to predict diameter inside bark from field measurement of diameter outside bark, or to predict diameter outside bark from diameter inside bark. For the latter, equation [4.4] is rearranged as follows:

$$[4.5] \qquad DOB = \frac{DIB + 0.234}{0.977}$$

where

DOB = Corresponding diameter outside bark (cm)
DIB = diameter inside bark at any point on the stem (cm).

The relationships between diameters inside and outside bark can also be used to compute bark thickness and, in conjunction with other equations, bark volume.

4.1.4. The tape model

The variable-exponent tape equation fitted the data very well (Figures 4.1 and 4.2). The resulted equation is given in Eq. [4.6].

$$[4.6]\, d = 1.136515 D^{0.883765}\, 1.002133^{D}\, X^{0.234755Z^2 - 0.053431\ln(z=0.001) + 0.205346\sqrt{Z} - 0.019375e^{z} + 0.260283(D/H)}$$

$$R^2 = 0.988 \qquad RMSE = 0.52\ (cm) \qquad n = 1826$$

where

$X = (1 - \sqrt{h/H}) / (1 - \sqrt{p})$
d = diameter inside bark (cm) at h
h = height above ground (m)
H = total tree height
D = diameter at breast height outside bark (cm)
$Z = h/H$
p = location of the inflection point, assumed to be at 22.5% of the total height above the ground
ln = the natural logarithm
e = base of the natural logarithm ($\cong$ 2.71828)

RMSE = root mean squared error.

Tape equation can be used to predict diameter of a tree at any height from ground using breast height diameter outside bark and total height of the tree. The use of tape equation for individual tree volume estimation has become an increasingly popular trend (e.g., Kozak 1988, Newnham 1992, Flewelling and Raynes 1993, Huang 1994). In this study, however, we did not directly apply the tape equation method to the estimation of total tree volume. It was only used to predict diameter at specific points of tree stem (e.g., 17.5 cm above ground) and height at a specific top diameter (e.g., 6.3 and 7.6 cm) for the purpose of calculating volume of specific section of stem or merchantable volume. Huang (1994) developed tape equations of the same type for major timber species of Alberta according to natural region groups. Significant differences were found among different natural region groups for each species studied in Alberta. If different regions of the same province require different growth/yield models, it may be unwise for us to use growth/yield models developed in other provinces. We should develop our own growth/yield models in Manitoba, and ideally, these models should also be ecologically based (i.e., taking climate and site types into account). Black spruce, due to its wide range of distribution and ecological adaptation, is an ideal candidate for investigating how ecological variables affect growth and yield and whether separate models are needed for different regions and/or site types. To develop such ecologically-based growth and yield model, additional data should be collected from black spruce stands on mineral soils with and without ground water influences.

4.1.5. The volume models

Tree volume was found closely related to either breast height diameter alone (Eq. [4.7], Figure 4.3) or the product of squared breast height diameter and total height of the tree (Eq. [4.8], Figure 4.4):

[4.7] $$V_{OB} = 0.996827(1 - e^{-0.066232D})^{4.342164}$$

$R^2 = 0.977$ $RMSE = 0.012\ (m^3)$ $n = 143$

where

V_{OB} = total volume (m^3) outside bark

D = breast height diameter outside bark (cm)

e = base of the natural logarithm ($\cong 2.71828$)

$RMSE$ = root mean squared error.

[4.8] $$V_{OB} = 0.945457(1-e^{-0.444122\frac{D^2H}{10000}})^{0.987110}$$

$R^2 = 0.994$ $SEE = 0.0064$ (m^3) $n = 143$

where

V_{OB} = total volume (m^3) outside bark

D = breast height diameter outside bark (cm)

H = total tree height (m)

e = base of the natural logarithm($\cong 2.71828$)

$RMSE$ = root mean squared error.

By applying the above equations, individual tree volume outside bark can be reliably predicted from dbh and total height.

Table 4.2. Prediction accuracy: Eq. [4.7] vs. Eq. [4.8].

Model	Residual range (m^3)	Mean % bias	Mean % error
1. Test based on all 143 trees			
Eq. [4.7]	-0.036 - 0.053	9.0	18.4
Eq. [4.8]	-0.027 - 0.021	3.5	6.8
2. Test based on 95 tree (DBH ≥ 9.1 cm)			
Eq. [4.7]	-0.036 - 0.053	-2.6	10.7
Eq. [4.8]	-0.027 - 0.021	-0.05	4.5

Residual = observed - predicted.

% bias = residual x 100 / observed.

% error = |residual| x 100 / observed.

A comparison of the two models is given in Table 4.2. As expected, Eq. [4.8] fitted data better than Eq. [4.7]. It appeared that larger estimation errors (under-estimation) are associated with smaller trees as indicated by positive bias (Table 4.2).

When only trees with breast height diameter $\geq$ 9.1 cm were considered, estimation improved for both models. Eq. [4.7] slightly over-estimated volume by an average of 2.6% while virtually no bias was found for Eq. [4.8]. If a maximum mean error of 10% was acceptable, Eq. [4.8] would be more than satisfactory. The estimation error by using Eq. [4.7] was slightly over 10%, which may be a good trade-off if height is not available or cost is high to obtain height measurements.

A significant relationship was found between volumes inside and outside bark (Eq. [4.9]):

[4.9]

$$V_{IB} = -0.001522 + 0.93792V_{OB}$$

$$R^2 = 0.999 \qquad SEE = 0.002354 \qquad n = 143$$

where

V_{IB} = total volume (m^3) inside bark
V_{OB} = total volume (m^3) outside bark
SEE = standard error of estimate.

By applying Eq. [4.9], total volume inside bark can be reliably predicted from total volume outside bark.

Significant relationships were also found between merchantable volume (inside bark) and total volume (outside bark) (Eqs. [4.10] and [4.11]). Merchantable volume in Eq. [4.10] is defined according to the provincial criteria (G. Becker, pers. comm.); merchantable volume in Eq. [4.11] is defined according to the criteria of Pine Falls Paper Company (V. Keenan, pers. comm).

[4.10]

$$V_M = -0.013212 + 0.907935V_{OB}$$

$$R^2 = 0.998 \qquad SEE = 0.00282 \qquad n = 95$$

[4.11]

$$V_M = -0.009171 + 0.905160V_{OB}$$

$$R^2 = 0.998 \qquad SEE = 0.003167 \qquad n = 77$$

where

V_{OB} = total volume (m^3) outside bark
V_M = merchantable volume (m^3) inside bark

SEE = standard error of estimate.

Merchantable volume saved by using a stump height of 17.5 cm, instead of 30 cm depends on the size of the tree (Figure 4.5), and it can be predicted from breast height diameter using Eq. [4.12]:

[4.12] $$V_{save} = e^{-11.555643+2.084601 ln(D)}$$

$R^2 = 0.990$ $RMSE = 0.0001$ $n = 77$

where

V_{save} = volume saved by lower stump height from 30 to 17.5 cm.

D = breast height diameter (cm) outside bark

ln = natural logarithm

RMSE = root mean squared error.

When the saved volume was expressed as % of the merchantable volume (stump height = 30 cm), it ranged 1.5 to 5.5% depending on tree size although mostly between 2-3% (Figure 4.6).

4.2. Testing Existing Individual Tree Volume Models

4.2.1. The volume model used by Manitoba Forestry Branch

The merchantable volume equation used by Manitoba Forestry Branch was tested using 95 stem analysis trees sampled in our study, and the test results are given in Table 4.3. The provincial equation only provided predictions of merchantable volumes (dbh > 9.1 cm; top = 7.6 cm; stump height = 30 cm).

On average, the volume equation underestimated volume by 13.9%, with a range of -12.6% to +40.0%. The biased prediction was further demonstrated in Figure 4.7. The equation underestimated volume when tree was small and overestimated volume when tree was large. That is why a different formula was used by Manitoba Forestry Branch (Gerry Becker, pers. comm.) for trees with dbh $\geq$ 24.1 cm. For trees with dbh between 9.1 to 24.0 cm (i.e., in this test), the equation systematically underestimated volume with few (6 trees) exceptions (Figure 4.7). We believe this may be an indication of a lack of fit in the original model. It would be the case if a linear model, instead of a nonlinear one, was applied to estimate total volume in this study (Figure 4.4).

Table 4.3. The test results of the volume equation used by Manitoba Forestry Branch based on 95 stem analysis trees sampled from lowland black spruce stands in Manitoba Model Forest Area.

	Range	Mean	SD	CV
1. Tree information				
Total height (m)	7.3 - 20.7	13.5	3.1	0.23
DBH (cm)	9.1 - 24.0	14.0	3.3	0.24
M. volume (m3)	0.014 - 0.332	0.095	0.068	0.72
2. Test result				
Residual	-0.037 - 0.020	0.010	0.010	3.64
% bias	-12.6 - 40.0	13.9	8.8	0.63
% error	0.4 - 40.0	14.6	7.5	0.51

SD - standard deviation.

CV - coefficient of variation.

Residual = observed - predicted.

% bias. = residual x 100 / observed.

% error = |residual| x 100 / observed.

Since negative and positive residuals may cancel each other, absolute residuals were also compared, and an average error of 14.6% was found. If a maximum mean error of 10% was acceptable, the provincial volume equation would not be appropriate for estimating merchantable volume of lowland black spruce stands in Manitoba Model Forest area.

4.2.2. The volume model used by Repap Manitoba Inc.

The total volume equation used by Repap Manitoba Inc. was tested using the 143 trees sampled in our study, and the test results are given in Table 4.4. On average, the volume equation underestimated volume by 14.8%, with a range of -4.5% to +45.2%. The biased prediction was further demonstrated in Figure 4.8. Except for four trees, volume was systematically underestimated up to 20% when tree volume > 0.1 m^3. Even larger underestimations occurred when tree volume < 0.1 m^3 (Figure 4.8). This bias may be due to the fact that the original model was developed based on data from upland black spruce stands, and uplands and lowlands may require different volume models. Currently, lowland black spruce has not been harvested in Repap's area (Fiona Donald, pers. comm.).

Since negative and positive residuals may cancel each other, absolute residuals were also compared and an average error of 15.0% was found for the equation If a maximum mean error of 10% was acceptable, the equation would not be appropriate to estimate total volume of black spruce trees from lowland black spruce stands.

Table 4.4. Testing the total volume equation used by Repap Manitoba using those stem analysis trees sampled from lowland black spruce stands in Manitoba Model Forest Area.

	Min.	Mean	SD	CV
1. Test based on all 143 trees				
Residual	-0.0100 - 0.0459	0.0094	0.0093	1.00
% bias	-4.5 - 45.2	14.8	8.1	0.54
% error	1.0 - 45.2	15.0	7.8	0.52
2. Test based on 95 tree (DBH > 9.1 cm)				
Residual	-0.0100 - 0.0459	0.0128	0.0097	0.76
% bias.	-4.5 - 31.2	11.7	5.9	0.50
% error	1.0 - 31.2	11.9	5.4	0.45

SD - standard deviation.

CV - coefficient of variation.

Residual = observed - predicted.

% bias. = residual x 100 / observed.

% error = |residual| x 100 / observed.

Since only trees above certain diameters would be harvested, the volume prediction equation was further assessed using only those 95 trees with DBH > 9.1 cm (Table 4.4). Although volume predictions were improved, both the bias and the mean error were still higher (11.7 and 11.9%, respectively) than 10%.

We further tested the performance of the model based on 30 trees sampled from upland black spruce stands. These data were not used to develop growth/yield models reported in the study. A comparison of our model (Eq. [4.8]) and the model used by Repap Manitoba is given in Table 4.5. Although both models underestimated tree volume, our model performed much better than the model used by Repap. The mean bias and error of predictions derived from Repap's model were about 3 times of that derived from our model (Table 4.5), which also exceeded 10%. It appears that the model performed equally for both lowland and upland trees. We believe this bias may be due to the lack of fit in the original model. Therefore, an improved model can be developed. It would be interesting to see, however, what would be the result if tested by data collected in Repap's area.

Table 4.5. Prediction accuracy: Eq. [4.8] vs. the model used by Repap Manitoba. Test based on 30 stem analysis trees sampled in the study area. Heights of the sampled trees are between 14.3 to 20.8 m; breast height diameters of those sampled trees are between 13.0 to 22.7 cm.

Model	Residual range (m^3/ha)	Mean % bias	Mean % error
Eq. [4.8]	-0.0147 - 0.0584	3.8	5.3
Repap model	0.0034 - 0.0738	14.0	14.0

Residual = observed - predicted.

% bias = residual x 100 / observed.

% error = |residual| x 100 / observed.

4.3. Height and Site Index Models

4.3.1. The height model

The total height of dominant black spruce trees in relation to site index and breast height age was quantified by Eq. [4.13]:

[4.13] $$H = 1.3 + 22.1825SI^{0.2753}(1 - e^{-0.003932A})^{2.0226SI^{0.3802}}$$

$$R^2 = 0.978 \quad RMSE = 0.61\ (m) \quad n = 266$$

where

H = total tree height (m)

SI = Site index (m)

A = breast height age (years)

e = base of the natural logarithm($\cong 2.71828$)

$RMSE$ = root mean squared error.

As indicated by the residual plot (Figure 4.9), the above equation fitted the data very well. Given a site index, Eq. [4.13] can be used predict height at any breast height age. For any measured height and breast height age, site index can be calculated from Eq. [4.13] through an iteration procedure.

4.3.2. Site index model

To avoid using an iteration procedure in calculating site index from a given height and breast height age, site index was expressed as a function of height and breast height age (Eq. [4.14]):

[4.14] $$SI = 0.04841H^{0.6575}(1 - e^{-0.000318A})^{-0.72326H^{0.1067}}$$

$$R^2 = 0.916 \quad RMSE = 0.86\ (m) \quad n = 245$$

where

SI = Site index (m)

H = total tree height (m)

A = breast height age (years)

e = base of the natural logarithm ($\cong 2.71828$)

$RMSE$ = root mean squared error.

As indicated by the residual plot (Figure 4.10), the above equation fitted the data very well. The solution of Eq. [4.14] for a given height and age will provide an estimate of site index for peatland black spruce stands in the study area.

As both height and site index model were developed based on data from lowland stands, the models may not necessarily applicable to uplands. Even for application to lowlands, application should be done with caution if it is beyond the range of the data of this study. This note of caution is important as our data set is relatively small. Basic statistics of the data used in our study are given in Table 4.6.

Table 4.6. Summary of selected variables used in developing height and site index models in the study.

Stem attribute	Range	Mean.	SD	CV
Total height (m)	9.51 - 19.47	14.37	2.35	0.16
Total age (year)	60 - 191	125	42	0.34
Breast height age (year)	47 - 173	106	39	0.37
Site index (m)	3.65 - 15.35	8.41	3.34	0.40

SD - standard deviation.

CV - coefficient of variation.

We did not compare our models to other developed height and site index models for black spruce because such models were not developed in Manitoba. Although such models were developed elsewhere (e.g., Ontario) (Payandeh 1977, 1978), these earlier models used total age instead of breast height age as a predictor, thus it is difficult to compare with our models. Breast height age is preferred by most researchers in developing height and site index models because competition, not site quality, is believed to be the major factor affecting height growth before trees reach 1.3 m (e.g., Clutter *et al.* 1983, Davis and Johnson 1987)

4.4. Stand Volume(Yield) Models

4.4.1. Total volume models

Basal area was found to be the best predictor of yield of lowland black spruce stands (Eq. [4.15]):

[4.14] $$Y = e^{0.660981+1.318966\ln(BA)}$$

$R^2 = 0.976$ $SEE = 16.89\ (m^3)$ $n = 37$

where

Y = stand volume (m3/ha)

BA = basal area (m2/ha)

e = base of the natural logarithm

ln = the natural logarithm.

SEE = standard error of estimate.

As further illustrated in the graphic presentation (Figure 4.11), the above equation fitted the data very well.

If average stand height were available, yield prediction can be further improved by applying Eq. [4.16]:

[4.15] $$Y = 752.59(1 - e^{-0.001011(BAxHT)})^{0.945276}$$

$R^2 = 0.994$ $SEE = 8.82\ (m^3)$ $n = 37$

where

Y = total stand volume (m3/ha)

BA = basal area (m2/ha)

HT = mean stand height (m)

SEE = standard error of estimate.

As further illustrated in the graphic presentation (Figure 4.12), the above equation fitted the data very well.

Comparisons on the prediction from the above two models are given in Table 4.7. The mean biases for both models were very small, indicating the selected models are correct for the data. As expected, the model based on both basal area and mean stand height yielded an improved estimate of stand volume compared to the model based only on basal area. However, we considered that the resulting improvement was only marginal (1.6% improvement in mean error). Whether such an improvement justifies the cost for determining mean stand height would be a decision that has to be made by model users.

Table 4.7. Prediction accuracy (mean bias, % bias, error, and % error) of Eq. [4.14] and Eq. [4.15].

Model	Residual range (m^3/ha)	Mean bias (m^3/ha)	Mean error (m^3/ha)	% bias	% error
Eq. [14]	-44.0 - 44.6	-0.16	10.8	-0.10	6.5
Eq. [15]	-15.9 - 19.7	-0.25	6.4	-0.15	3.9

Residual = observed - predicted.

Mean bias = mean residual.

Mean error = mean absolute residual.

% bias = mean residual x 100 / mean observation

% error = mean absolute residual x 100 / mean observation.

A summary of stand attributes of the data used in the above two models is given in Table 4.8. Without independent tests, the above models should not be applied beyond the range of the data or to a different population (e.g., upland stands).

Table 4.8. Summary of selected stand attributes of lowland black spruce stands sampled in the study (n = 37).

Stand attribute	Min.	Mean	SD	CV
Mean height (m)	2.97 - 12.35	7.42	2.63	0.39
Basal area (m2/ha)	2.01 - 52.9	27.82	15.06	0.54
Volume (m3/ha)	5.38 - 365.92	165.95	106.90	0.64

SD - standard deviation.

CV - coefficient of variation.

It should be noted that both site index and stand age are not good predictors of total stand volume based on our data. The result is difficult to be explained biologically because both site quality and stand age are widely acknowledged as important factors affecting yield (e.g., Clutter *et al.* 1983, Davis and Johnson 1987). A closer examination of the data revealed that site index was closely related to stand age (r = -0.83, p = 0.000), i.e., older stands sampled are of lower site quality. Therefore, the insignificant effects of both site index and breast height age is likely caused by multicollinearity.

4.4.2. The merchantable volume models

Although basal area remained as the most significant predictor of merchantable stand volume, site index and age (mean breast height age of dominant trees) became significant predictors as well (Eqs. [4.17] and [4.18]). Merchantable volume in Eq. [4.17] and [4.18] follows the criteria of Manitoba Forestry Branch and Pine Falls Paper Company, respectively.

[4.17] $$MSV = e^{0.870845+1.203074\ln(BA)+0.173463\ln(SI)-15.275767/A}$$

$$R^2 = 0.996 \quad RMSE = 5.01\ (m^3) \quad n = 22$$

where

MSV = merchantable stand volume (m^3/ha)
BA = basal area (m^2/ha)
SI = site index (m)
A = breast height age (year)
e = base of the natural logarithm ($\cong 2.71828$)
ln = the natural logarithm
$RMSE$ = root mean squared error.

[4.18] $$MSV = e^{1.125444+1.100395\ln(BA)+0.299380\ln(SI)+-24.220457/A}$$

$$R^2 = 0.991 \quad RMSE = 7.87\ (m^3) \quad n = 22$$

where

MSV = merchantable stand volume (m^3/ha)
BA = basal area (m^2/ha)
SI = site index (m)
A = breast height age (year)

e = base of the natural logarithm ($\cong 2.71828$)
ln = the natural logarithm
$RMSE$ = root mean squared error.

A summary of variables used in Eqs. [4.17] and [4.18] is given in Table 4.9.

Table 4.9. Summary of selected stand attributes of lowland black spruce stands sampled in the study (n = 22).

Variable	Range	Mean	SD	CV
For both equations:				
Site index (m)	3.65 - 15.35	8.56	3.34	0.39
Breast height age (year)	47 - 173	103	38	0.37
For Eq. [4.17]:				
Basal area (m^2/ha)1	10.08 - 46.46	26.67	10.48	0.39
Volume (m^3/ha)	51.93 - 300.47	153.54	72.52	0.47
For Eq. [4.18]:				
Basal area (m^2/ha)1	3.80 - 38.61	19.40	10.99	0.57
Volume (m^3/ha)	17.93 - 275.23	119.51	77.98	0.65

SD - standard deviation.
CV - coefficient of variation.

Graphical presentations of Eqs [4.17] and [4.18] are given in Figures 4.13 and 4.14, respectively. The near perfect linear relationship between measured and predicted merchantable stand volumes indicate that both models are fitted very well to the data. Therefore, stand volume can be reliably predicted by measuring breast height diameters of trees with defined minimum size and breast height ages and total heights of three dominant trees on a sample plot. Breast height diameters are needed in calculating basal area; breast height ages and total heights of three dominant trees are needed in calculating site index and mean breast height age.

Figure 4.1. The linear relationship between measured and predicted diameters (cm). Tape model (Eq. [4.6]) was used to predict diameter at any height of a tree.

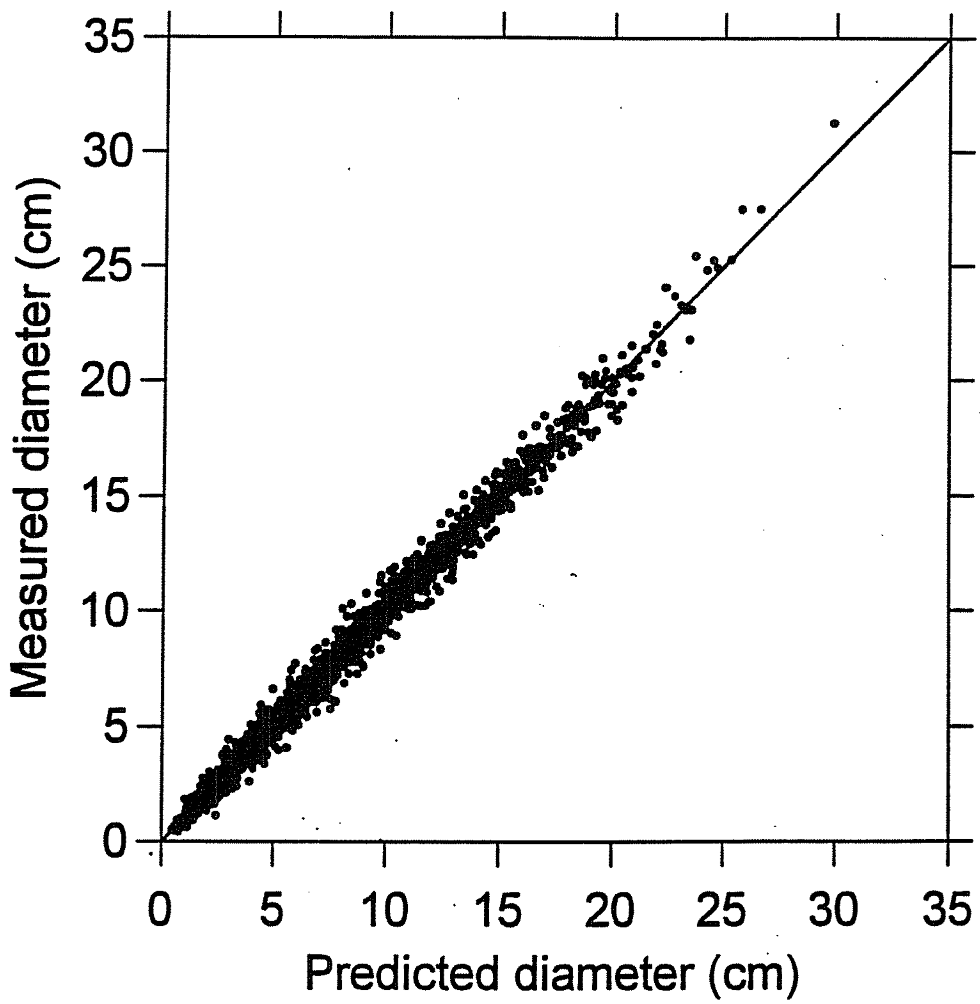


Figure 4.2. The residual plot of Eq. [4.6]. Estimate (cm) = predicted diameter from Eq. [4.6] and residual (cm) = measured diameter - predicted diameter.

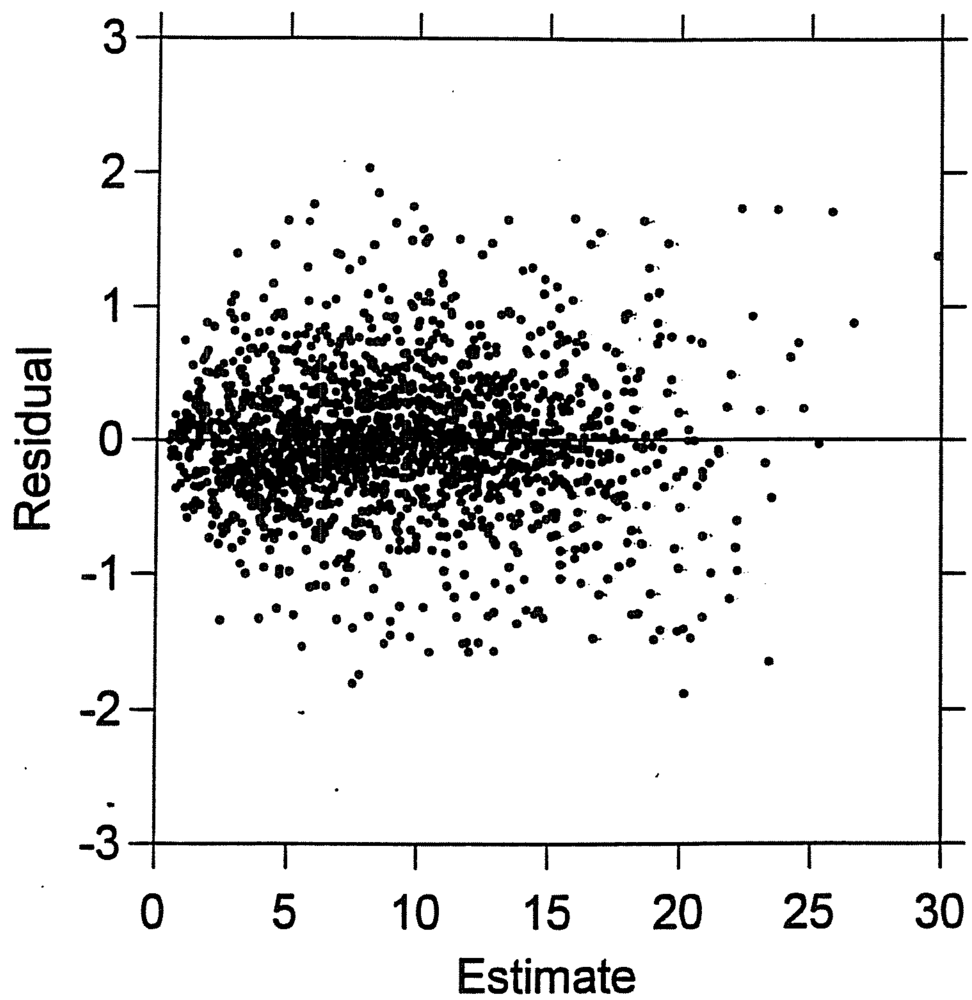


Figure 4.3. Scatterplot of total volume (m^3, outside bark) vs. breast height diameter (cm, outside bark). The curve is drawn according to Eq. [4.7].

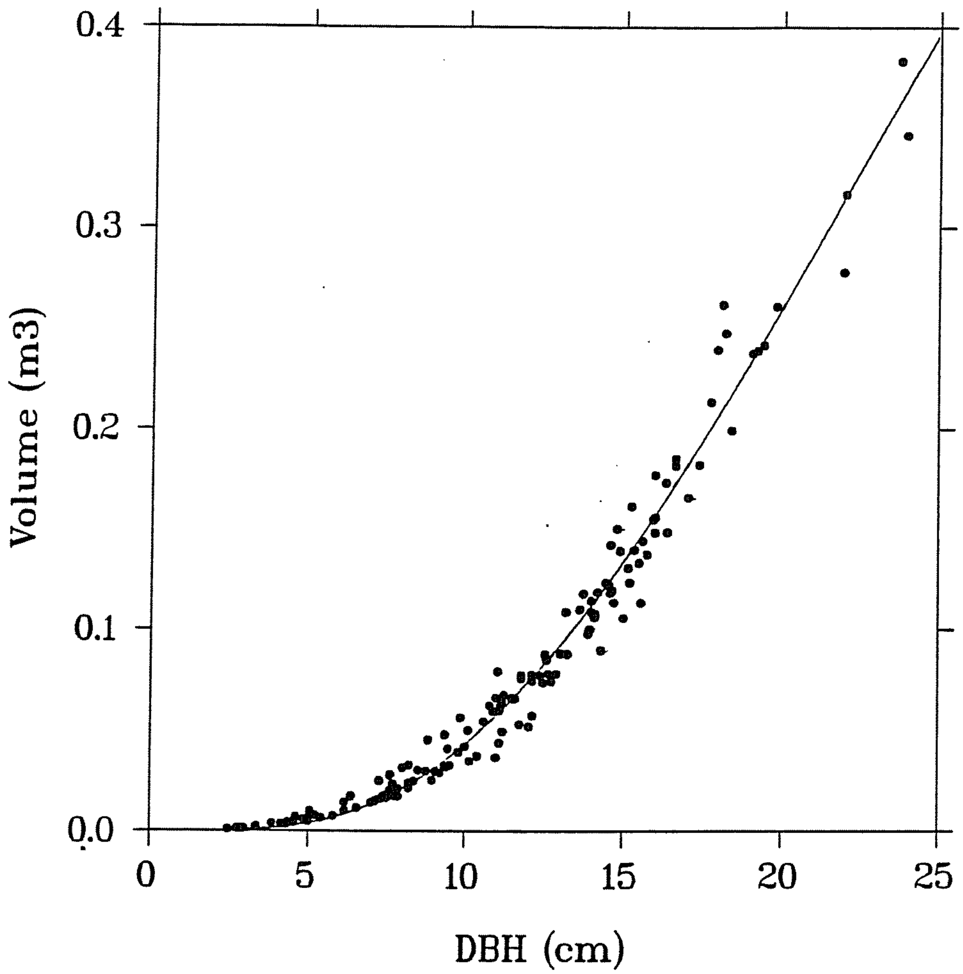


Figure 4.4. Scatterplot of total volume (m^3, outside bark) vs. dimension index (m^3). The curve is drawn according to Eq. [4.8]. Dimension index = squared breast height diameter x total height.

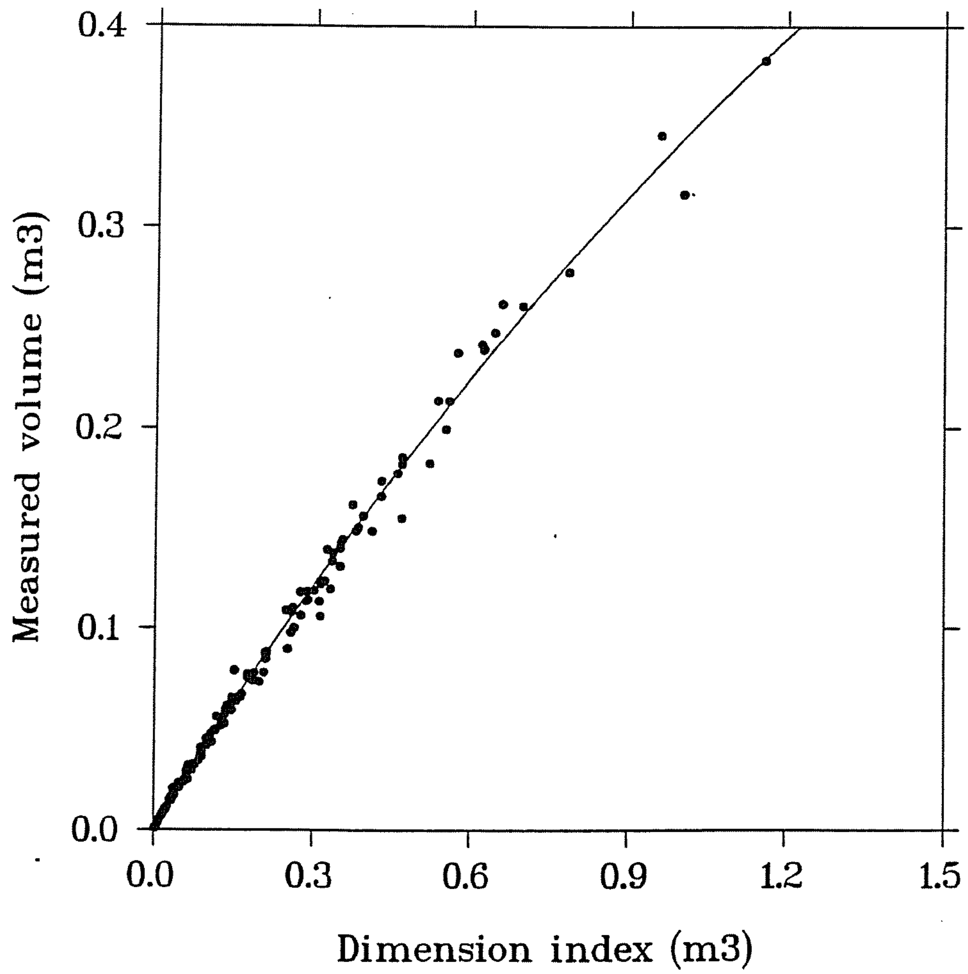


Figure 4.5. Scatterplot of the saved volume (m^3, inside bark) vs. breast height diameter (cm, outside bark). The curve is drawn according to Eq. [4.12]. The saved volume is the volume of the stem section between 17.5 and 30 cm.

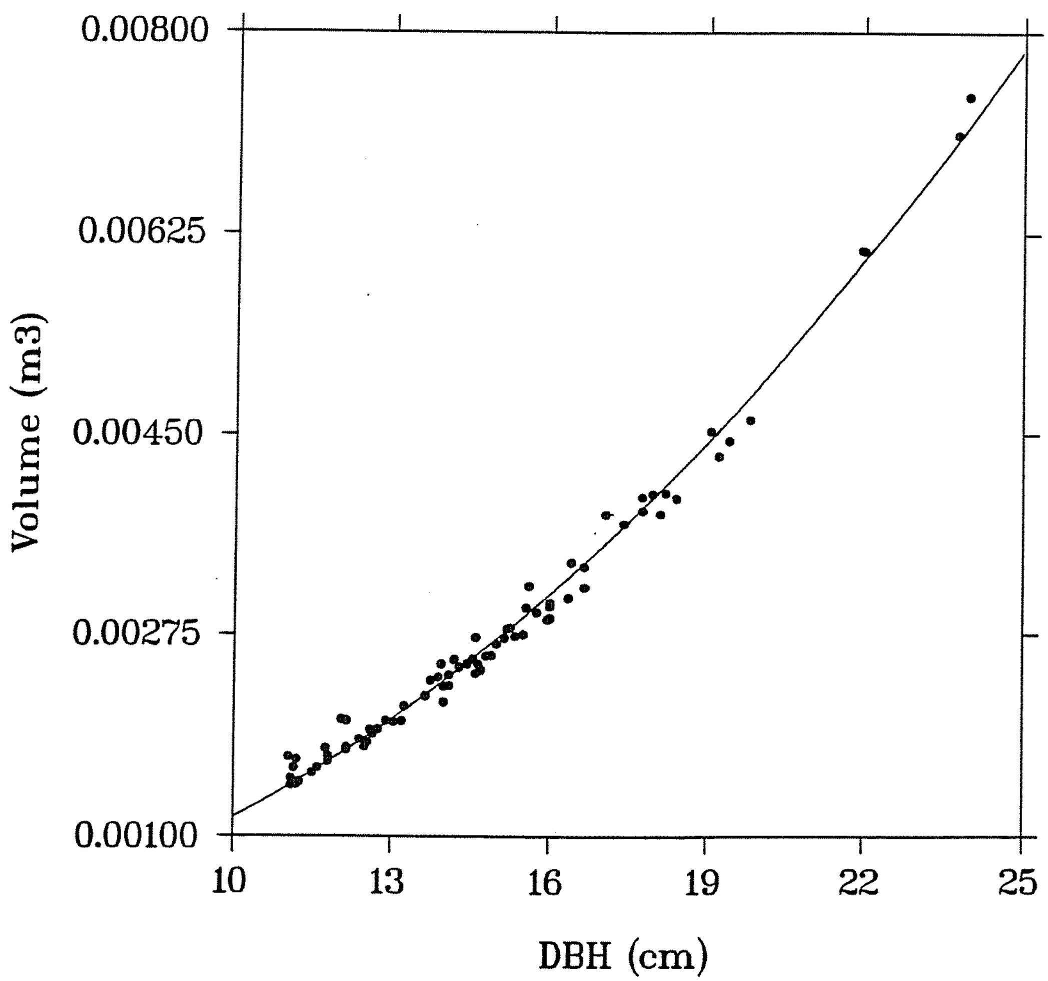


Figure 4.6. Scatterplot of % volume saved vs. merchantable volume (stump height = 30 cm). % volume saved = (the volume of the stem section between 17.5 and 30 cm / the merchantable volume) x 100.

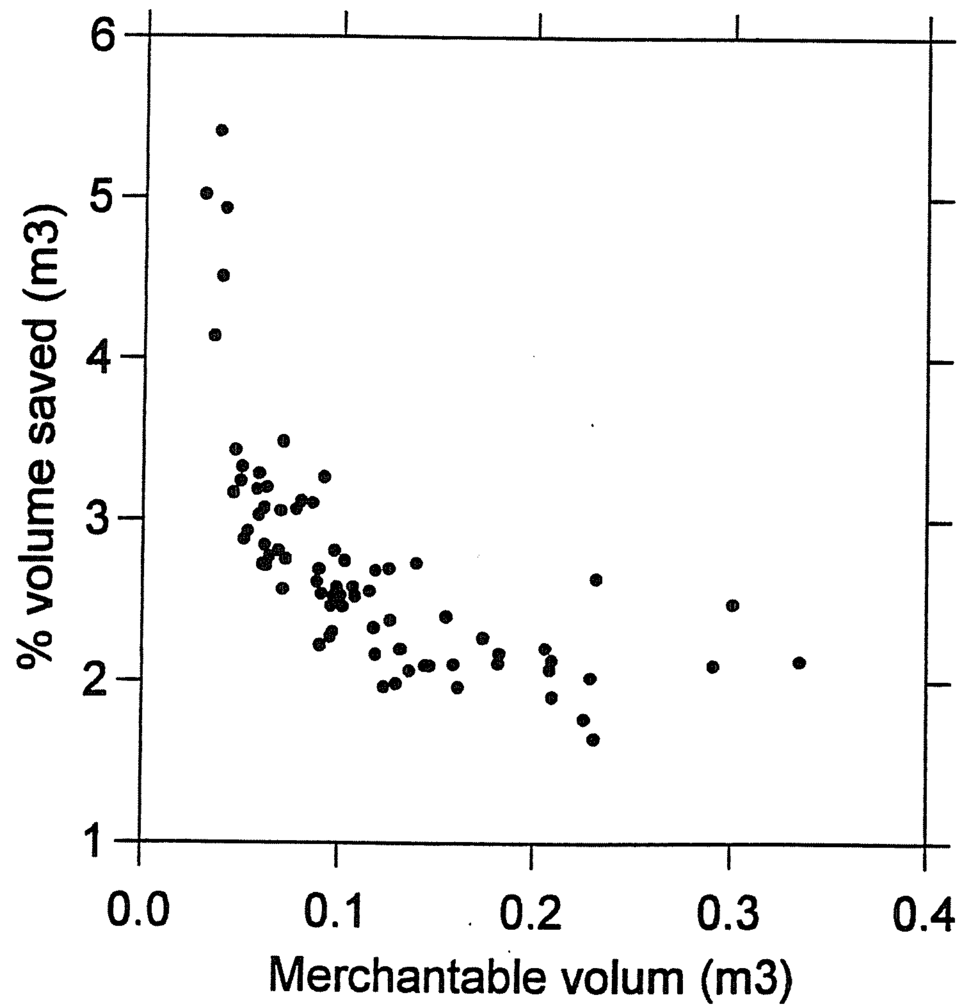


Figure 4.7. Scatterplot of % bias vs. observed volume (m^3) with a reference line that parallels to x axis and goes through y = 0. % bias = (observed volume - predicted volume) x 100 / observed volume. Volume predictions are made by using Eq. [3.13].

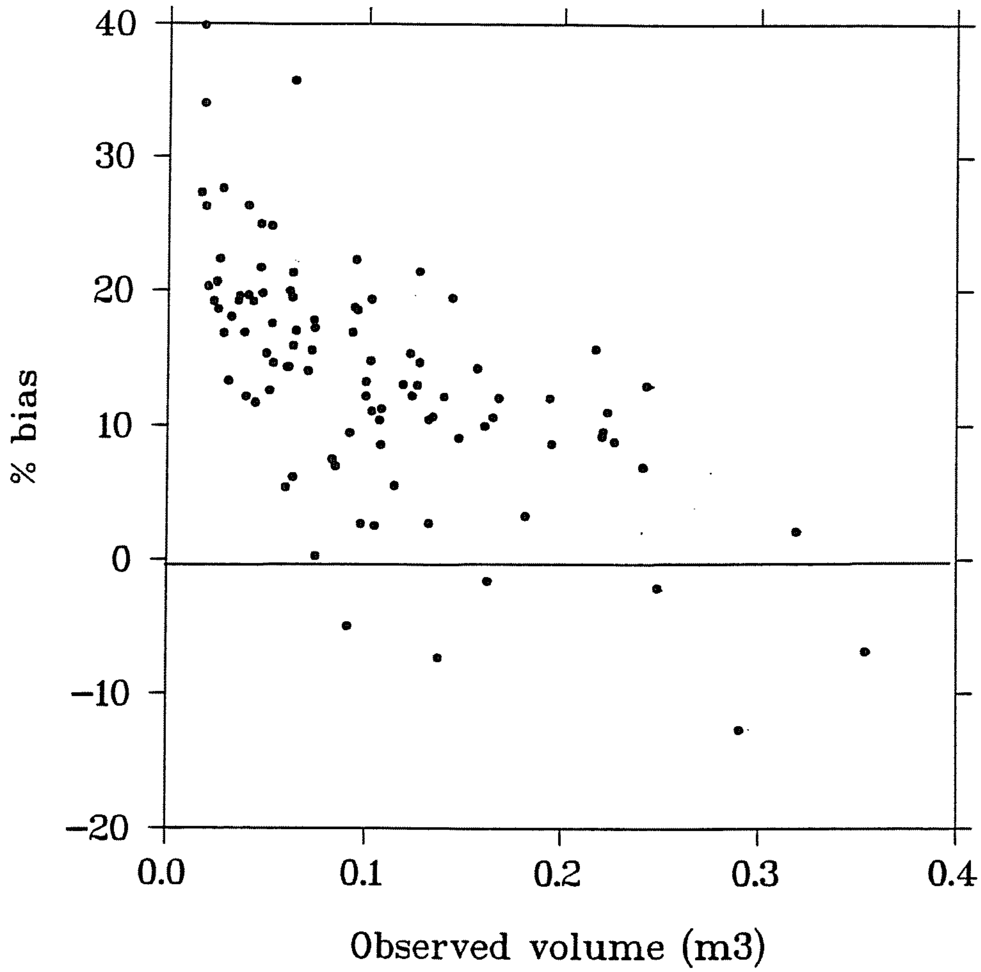


Figure 4.8. Scatterplot of % bias vs. observed volume (m^3) with a reference line that parallels to x axis and goes through y = 0. % bias = (observed volume - predicted volume) x 100 / observed volume. Volume predictions are made by using Eq. [3.14].

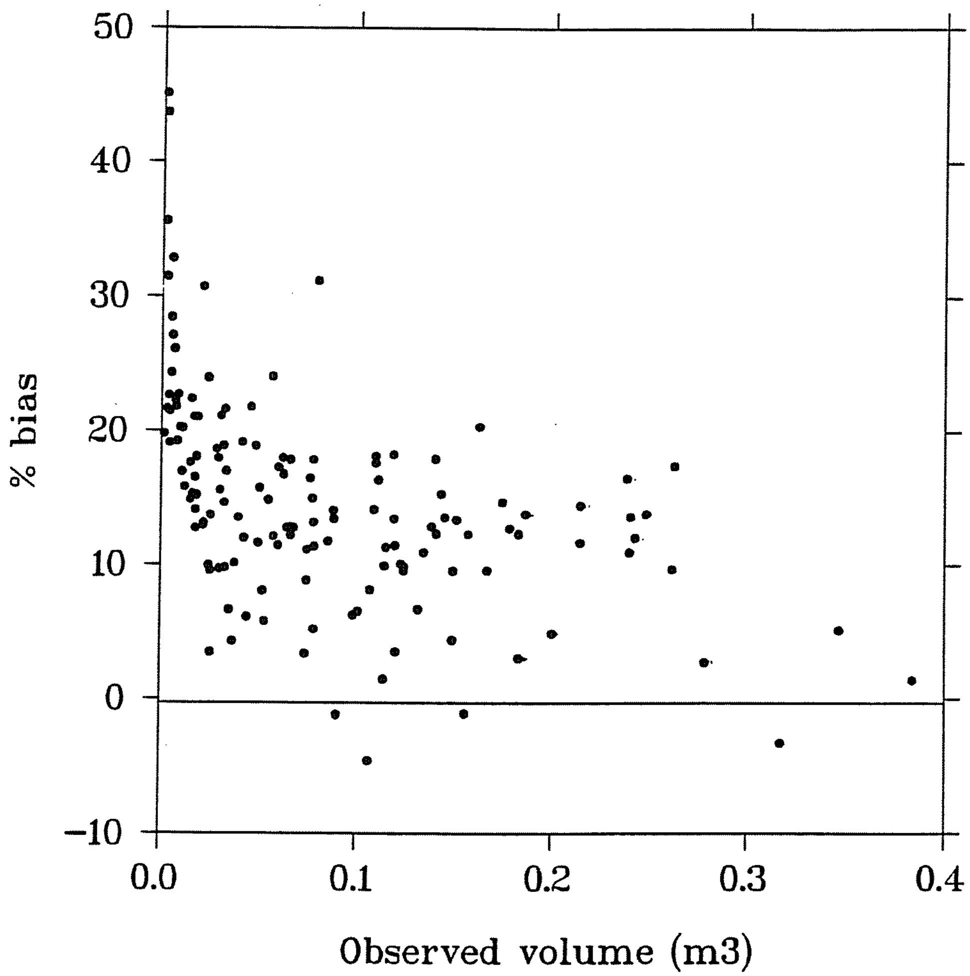


Figure 4.9. The residual plot of Eq. [4.13]. Estimate (m) = predicted height from Eq. [4.13] and residual (m) = measured height - predicted height.

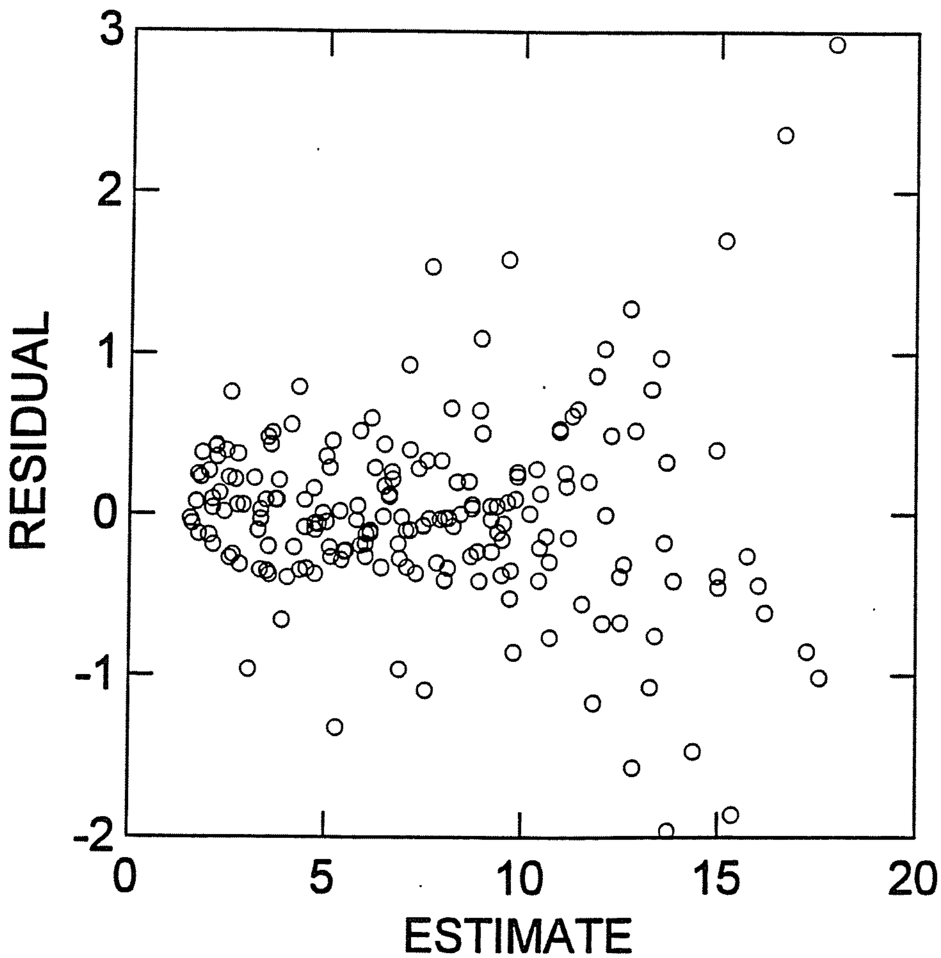


Figure 4.10. The residual plot of Eq. [4.14]. Estimate (m) = predicted site index from Eq. [4.14] and residual (m) = measured height - predicted height.

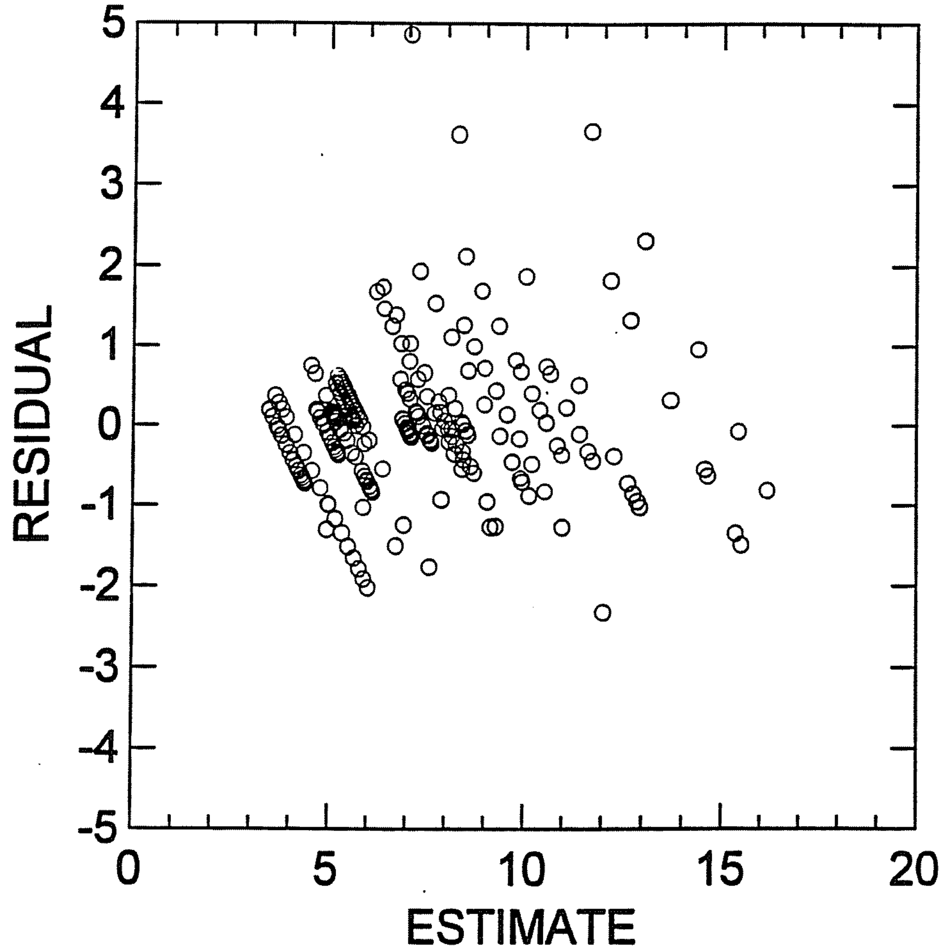


Figure 4.11. Scatterplot of total stand volume (m^3/ha) vs. basal area (m^2/ha). The curve is drawn according to Eq. [4.15].

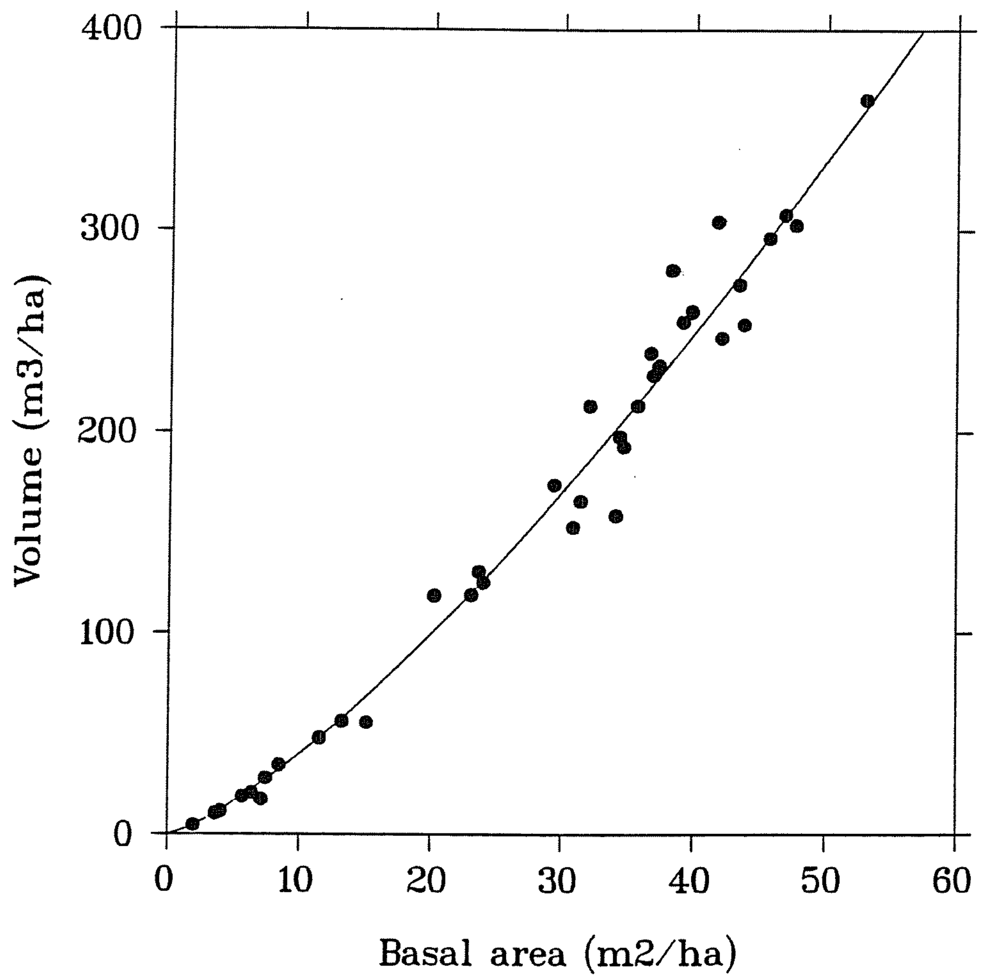


Figure 4.12. Scatterplot of total stand volume (m^3/ha) vs. product of basal area and mean stand height (m^3/ha). The curve is drawn according to Eq. [4.16].

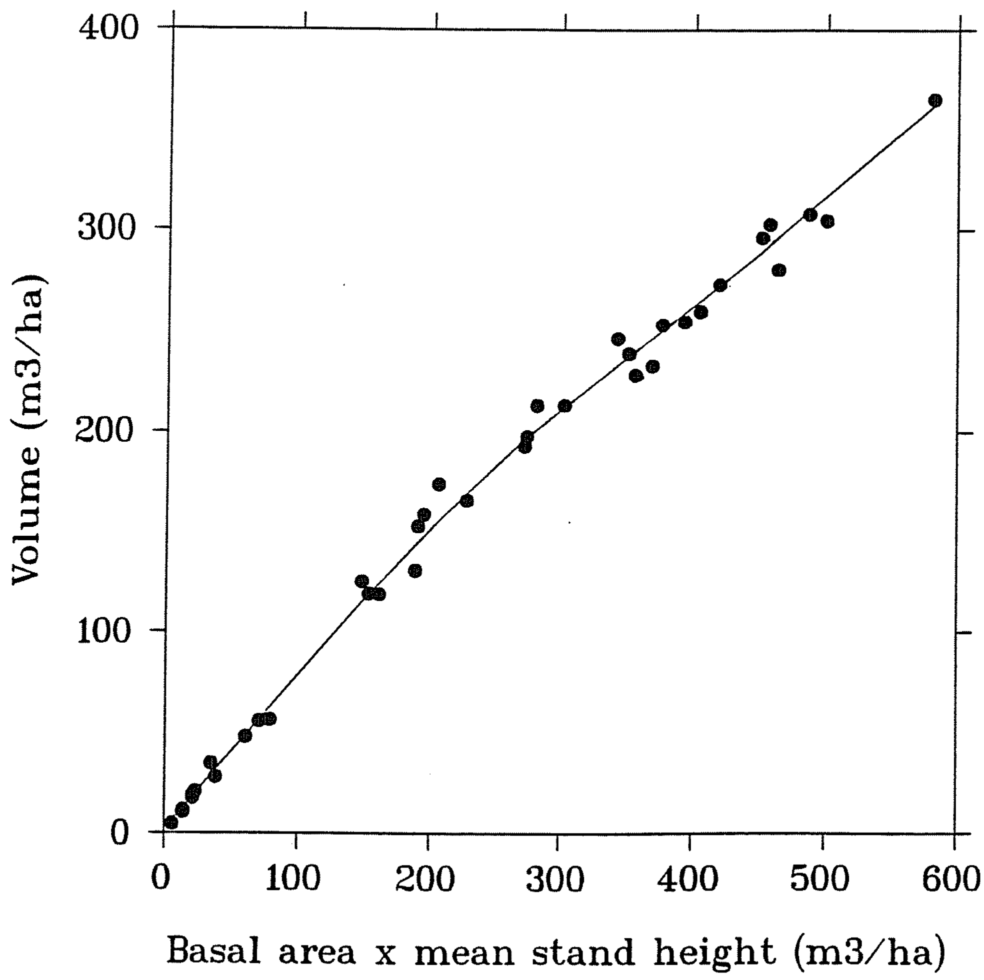


Figure 4.13. The linear relationship between measured predicted merchantable stand volume (m^3). Eq. [4.17]) was used to predict merchantable stand volume.

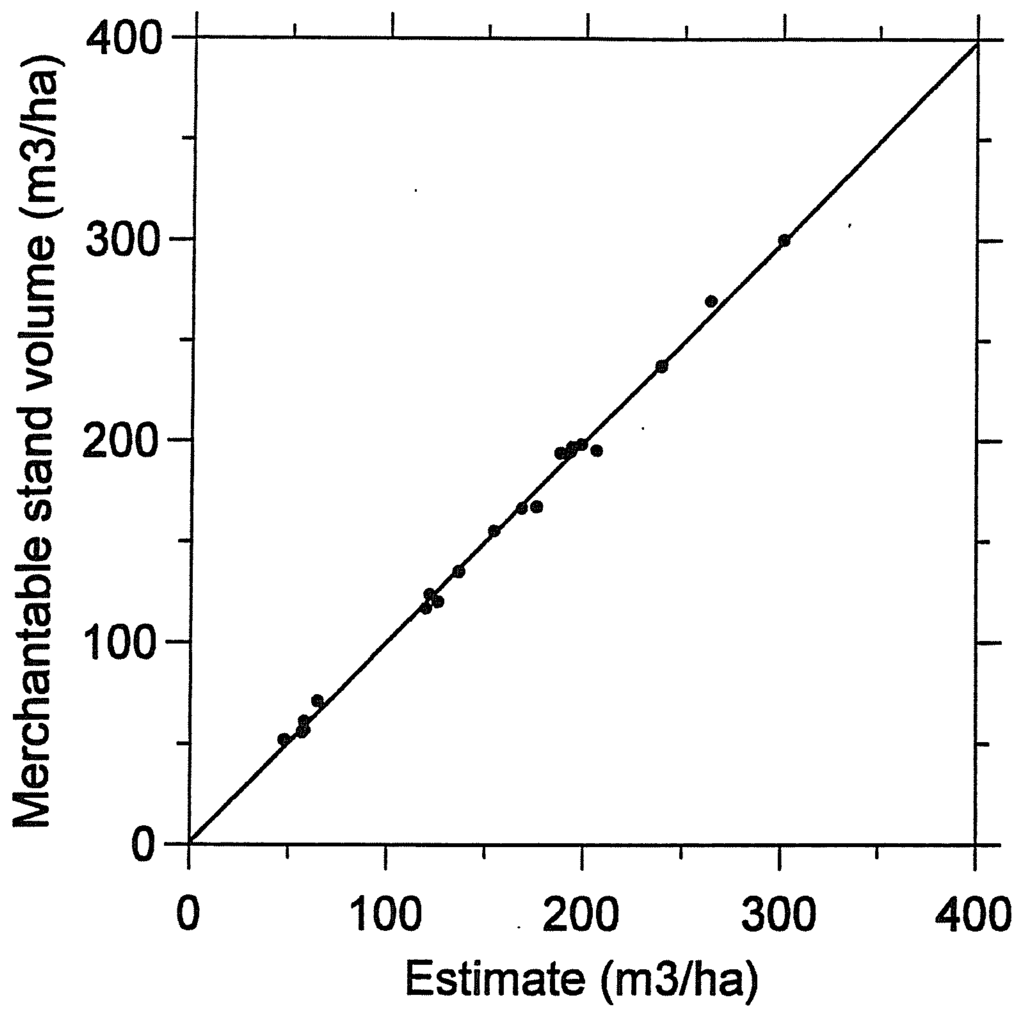
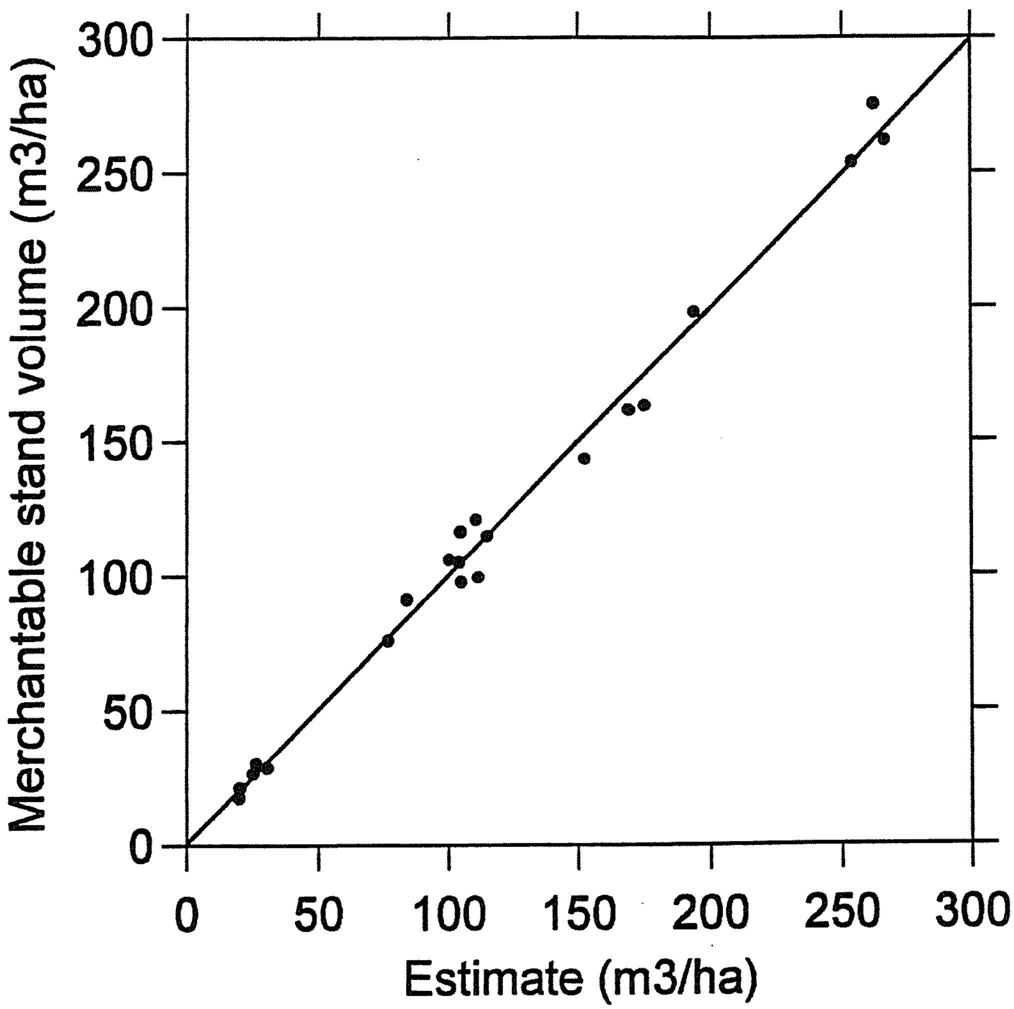


Figure 4.14. The linear relationship between measured predicted merchantable stand volume (m^3). Eq. [4.18]) was used to predict merchantable stand volume.



5. CONCLUSIONS AND RECOMMENDATIONS

The study provided a set of preliminary growth/yield models that can be applied to lowland black spruce stands in Manitoba Model Forest area. These models included various individual tree models, height/site index models, and stand yield models. The study not only filled a data gap of lowland black spruce growth and yield, but also provided basic methodologies to future growth/yield studies in Manitoba.

Since the models developed in the study was based only on a relatively small data set obtained from one geographical region (i.e., Manitoba Model Forest area) and one site type (i.e., organic soils), it is advised that these models may not necessarily apply to other regions or other site type. Independent testing is recommended should such an application be needed.

Testing existing models of individual tree volume revealed some problems with bias and error. If only a maximum of 10% error were allowed, both tested models are not satisfactory in applying to volume estimation. In addition to relatively large error, both model are biased (underestimating volume). Furthermore, a changing trend of residuals with estimated volume was found, suggesting a lack of fit in the original models. Further improvement of these models or developing new models are recommended.

Given the wide ecological amplitude of black spruce, we recommend further study should be conducted in the same study area to collect data from black spruce stands associated with other types of soils. The new data together with data obtained in this study as well as other sources (e.g., temporary or permanent sample plots) can be used to develop local growth/yield models that will provide the best possible prediction for black spruce growing on any site conditions in the study area.

Considering the importance of the species to the province, we further recommend that more data should be collected across the province in order to develop ecologically-based growth/yield models that can apply to any type of stands in any geographical region. Such a study may need coordinated effort between government agencies and industries.

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